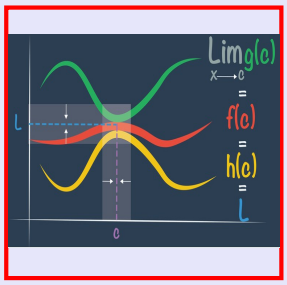
Math 261

Spring 2021

Lecture 39

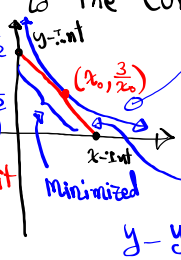


Find the shortest possible length of a line segment in QI that is tangent to the curve $y = \frac{3}{x}$ at some point.

$d = \sqrt{x\text{-Int}^2 + y\text{-Int}^2}$

$d = \sqrt{(2x_0)^2 + \left(\frac{6}{x_0}\right)^2}$

Minimize it



$y' = \frac{-3}{x^2}$

$m = \frac{-3}{x_0^2}$

$y - y_1 = m(x - x_1)$

$y - \frac{3}{x_0} = \frac{-3}{x_0^2}(x - x_0)$

$y = \frac{-3}{x_0^2}x + \frac{3}{x_0} + \frac{3}{x_0}$

$\boxed{y = \frac{-3}{x_0^2}x + \frac{6}{x_0}}$

x-Int $\quad y = 0$

$\frac{\cancel{3}}{\cancel{x_0^2}}x = \frac{\cancel{6}^{2}}{\cancel{x_0}}$

$x_0 \qquad x = 2x_0$

$S(x) = 4x^2 + \frac{36}{x^2}$

$S'(x)$

$S''(x) > 0$ CU

Min.

distance $2\sqrt{6}$

$f(x) = 4x^2 + \frac{36}{x^2}$ $\qquad f'(x) = 8x - \frac{72}{x^3}$

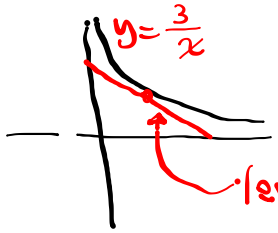
$f''(x) = 8 + \frac{216}{x^4} > 0 \therefore$ C.U.

$f'(x) = 0 \qquad 8x - \frac{72}{x^3} = 0 \qquad 8x^4 - 72 = 0$

$x^4 = 9 \qquad x = \sqrt{3}$ in QI

$d = \sqrt{4x^2 + \frac{36}{x^2}} \Rightarrow d = \sqrt{4(\sqrt{3})^2 + \frac{36}{(\sqrt{3})^2}} = \sqrt{12+12} = \sqrt{24}$

$= \sqrt{4 \cdot 6} = \boxed{2\sqrt{6}}$ ✓



length is min $2\sqrt{6}$.

Use Newton's Method to estimate $\sqrt[5]{25}$.

$\sqrt[5]{25} = 25^{\frac{1}{5}} = 25^{.2} \approx \boxed{1.904}$ From Calc.

Let $x = \sqrt[5]{25}$

$x^5 = 25 \rightarrow x^5 - 25 = 0$

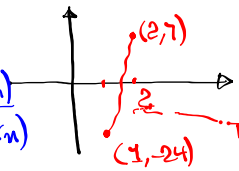
Let $f(x) = x^5 - 25$

Cont. Function

$f(1) = -24$

$f(2) = 7$

There is a Solution by I.V.T.



Newton's Eqn

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = x^5 - 25$

$f'(x) = 5x^4$

$x_{n+1} = x_n - \frac{x_n^5 - 25}{5x_n^4} = \frac{5x_n^5 - x_n^5 + 25}{5x_n^4}$

$x_{n+1} = \frac{4x_n^5 + 25}{5x_n^4}$

Let $x_1 = 2$

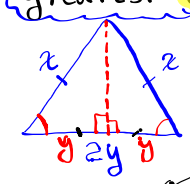
$x_2 = \frac{4(2)^5 + 25}{5(2)^4} = \frac{153}{80} = 1.9125$

$x_3 = \frac{4(1.9125)^5 + 25}{5(1.9125)^4} \approx 1.9037$

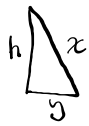
our answer

Double-Check my Calculation.

Show that of all the iso'sceies triangle with given perimeter, the one with greatest area is equilateral.



$P = x + x + 2y = 2x + 2y$



$h^2 + y^2 = x^2$

$h^2 = x^2 - y^2$

$h = \sqrt{x^2 - y^2}$

$\text{Area} = \frac{bh}{2} = \frac{\cancel{2}y \cdot \sqrt{x^2-y^2}}{\cancel{2}}$

we know $P = 2x + 2y$

$2x = P - 2y$

$x = \frac{P}{2} - y$

$\text{Area} = y\sqrt{x^2 - y^2}$

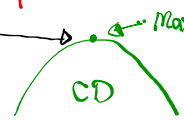
$A(y) = y\sqrt{\left(\frac{P}{2} - y\right)^2 - y^2}$

$A(y) = y\sqrt{\frac{P^2}{4} - Py + \cancel{y^2} - \cancel{y^2}}$

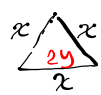
$A(y) = y\sqrt{\frac{P^2}{4} - Py}$

$A'(y) = 0$ → Max. Area

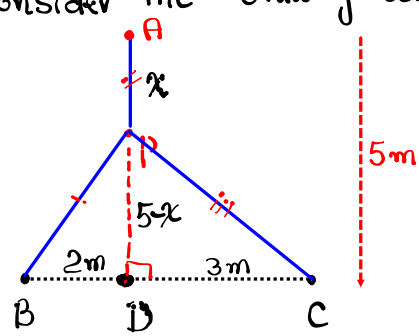
$A''(y) < 0$



Show $2y = x$



Consider the drawing below

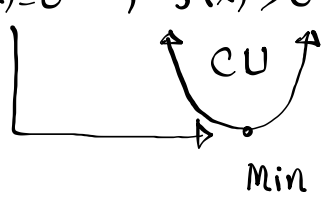


we want a cable to connect P to A, B, and C with minimum length.

Minimize $|AP| + |BP| + |CP|$.

$$f(x) = x + \sqrt{2^2 + (5-x)^2} + \sqrt{3^2 + (5-x)^2}$$

$f'(x) = ?$ $\quad f'(x) = 0$, $f''(x) > 0$



Class QZ
14

Use Calculus method to graph

$$f(x) = \frac{x-4}{x+2}$$